

**Federal State Autonomous Educational Institution of Higher Education "Moscow  
Institute of Physics and Technology  
(National Research University)"**

**APPROVED**

**Head of the Phystech School of  
Applied Mathematics and  
Informatics**

**A.M. Raygorodskiy**

**Work program of the course (training module)**

**course:** Advanced Graph Theory/Продвинутый курс по теории графов  
**major:** Applied Mathematics and Informatics  
**specialization:** Computer Science/Информатика  
Phystech School of Applied Mathematics and Informatics  
Chair of Discrete Mathematics  
**term:** 2  
**qualification:** Bachelor

Semester, form of interim assessment: 4 (spring) - Grading test

Academic hours: 60 AH in total, including:

lectures: 30 AH.

seminars: 30 AH.

laboratory practical: 0 AH.

Independent work: 75 AH.

In total: 135 AH, credits in total: 3

Author of the program: A.G. Ilinskiy, doctor of physics and mathematical sciences, associate professor

The program was discussed at the Chair of Discrete Mathematics 24.05.2023

## Annotation

This course can be considered as second part of the course "Combinatorics and Graphs" and is devoted to more complex topics in graph theory such as colourings, external graph theory, Ramsey-type problems, labelled trees and hyper graphs.

### 1. Study objective

#### Purpose of the course

mastering the main modern methods of graph theory.

#### Tasks of the course

- mastering by students of basic knowledge (concepts, concepts, methods and models) in graph theory;
- acquisition of theoretical knowledge and practical skills in graph theory;
- providing advice and assistance to students in conducting their own theoretical research in graph theory.

### 2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
Gen.Pro.C-2 Use modern IT and software tools to perform professional tasks in compliance with information security requirements	Gen.Pro.C-2.2 Apply numerical mathematical methods and use software applications for scientific problem-solving in professional settings
Pro.C-1 Assign, formalize, and solve tasks, develop and research mathematical models of studied phenomena and processes, systematically analyze scientific problems, obtain new scientific outcomes	Pro.C-1.2 Make hypotheses, build mathematical models of the studied phenomena and processes, evaluate the quality of the developed model

### 3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- fundamental concepts, laws, graph theory;
- modern problems of the corresponding sections of graph theory;
- concepts, axioms, methods of proofs and proofs of the main theorems in the sections included in the basic part of the cycle of graph theory;
- basic properties of the corresponding mathematical objects;
- analytical and numerical approaches and methods for solving typical applied problems of graph theory.

be able to:

- understand the task at hand;
- use your knowledge to solve fundamental and applied problems;
- evaluate the correctness of the problem setting;
- strictly prove or disprove the statement;
- independently find algorithms for solving problems, including non-standard ones, and analyze them;
- independently see the consequences of the results obtained;
- Accurately represent mathematical knowledge in topology both orally and in writing.

master:

- skills of mastering a large amount of information and solving problems (including complex ones);
- skills of independent work and mastering new disciplines;
- culture of setting, analyzing and solving mathematical and applied problems that require the use of mathematical approaches and methods for their solution;
- the subject language of topology and the skills of competently describing the solution of problems and presenting the results obtained.

### 4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

#### 4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Metrics on graphs. Graph diameter, radius, center.	6	6		15
2	Graph colorings. Definitions and example applications.	6	6		15
3	Digraphs. Tournaments.	6	6		15
4	Ford—Fulkerson’s “Max-Flow-Min-Cut” theorem on flows in networks. Cuts and flows as an example of “combinatorial duality”.	6	6		15
5	Labelled root trees. Cayley’s formula. Labelled forests.	6	6		15
AH in total		30	30		75
Exam preparation		0 AH.			
Total complexity		135 AH., credits in total 3			

#### 4.2. Content of the course (training module), structured by topics (sections)

Semester: 4 (Spring)

##### 1. Metrics on graphs. Graph diameter, radius, center.

Various notions of vertex centrality. Extremal problems on graphs. Turán’s theorem. More extremal problems: paths, trees and 4-cycles.

##### 2. Graph colorings. Definitions and example applications.

Greedy coloring. Chromatic number.  
Simple bounds for the chromatic number. Brook’s theorem

##### 3. Digraphs. Tournaments.

Some similarities and differences between directed and undirected graphs. Kernels.

##### 4. Ford—Fulkerson’s “Max-Flow-Min-Cut” theorem on flows in networks. Cuts and flows as an example of “combinatorial duality”.

Combinatorial application of integer flows: Hall’s theorem on matchings in bipartite graphs. Menger’s theorem. Proof of Menger’s theorem via Ford—Fulkerson’s.

##### 5. Labelled root trees. Cayley’s formula. Labelled forests.

Ramsey-type theorems in graph theory and combinatorics (a brief overview). Basic Ramsey theorem for graphs. Bounds for Ramsey numbers: a lower bound via counting, an upper bound by binomial coefficient.

## **5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)**

Standard classroom.

## **6. List of the main and additional literature, that is necessary for the course (training module) mastering**

### Main literature

1. Дискретный анализ. Комбинаторика. Алгебра логики. Теория графов [Текст] : учеб. пособие для вузов / Ю. И. Журавлев, Ю. А. Флеров, О. С. Федько ; М-во образования и науки РФ, Моск. физ.-техн. ин-т (гос. ун-т) .— М. : МФТИ, 2012 .— 248 с.
2. Теория графов [Текст] : [учеб. пособие для вузов] / О. Оре ; пер. с англ. И. Н. Врублевской ; под ред. Н. Н. Воробьева .— 2-е изд., стереотип. — М. : Наука, 1980 .— 336 с.

### Additional literature

1. Сборник задач по дискретному анализу. Комбинаторика. Элементы алгебры логики. Теория графов [Текст] : учеб. пособие для вузов / Ю. И. Журавлев [и др.] ; М-во образования Рос. Федерации, Моск. физ.-техн. ин-т (гос. ун-т) .— 2-е изд. — М. : МФТИ, 2000, 2004 .— 100 с.

## **7. List of web resources that are necessary for the course (training module) mastering**

<http://dm.fizteh.ru/>

<http://www.mccme.ru/~anromash/courses/expanders-notes-2014.pdf>

## **8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)**

Multimedia technologies can be employed during lectures and practical lessons, including presentations.

## **9. Guidelines for students to master the course**

1. It is recommended to successfully pass test papers, as this simplifies the final assessment of the subject.
2. To prepare for the final certification in the subject, it is best to use the lecture materials.

**Assessment funds for course (training module)**

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## 1. Competencies formed during the process of studying the course

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## 2. Competency assessment indicators

As a result of studying the course the student should:

### know:

- fundamental concepts, laws, graph theory;
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## 3. List of typical control tasks used to evaluate knowledge and skills

Examples of home tasks

1. What is the maximum number of edges in a graph with 12 vertices with 3 connected components?
2. How many pairwise non-isomorphic 2-regular bipartite graphs on 16 vertices exist?
3. How many different 2-regular graphs on 16 vertices with fixed parts  $\{x_1, \dots, x_8\}$  and  $\{y_1, \dots, y_8\}$  exist?
4. What is the maximum number of vertices in a graph in which there is neither an independent set on three vertices nor odd cycles?
5. Find the smallest possible independence number of a graph on 6 vertices without triangles.

## 4. Evaluation criteria

1. Extension properties, connections between them. Definition of regular expanders, proof of existence. Applications: improving success in algorithms.
2. Bipartite expanders, existence. Applications: asymptotically good codes, superfast data storage.
3. Spectral theory of expanders. Algebraic expanders. Lower bound for the second eigenvalue. Existence theorems.

4. Lemma on mixing. Edge extension theorem.
5. Connection between combinatorial and algebraic expanders. Random walks over expanders. Applications.
6. Substitution product, tensor product, zigzag product of graphs. Spectral properties.
7. Recursive constructions of expanders. On the "explicit" assignment of graphs.
8. Counts of Ramanujan, construction of Margulis.
9. Applications: Reingold's algorithm for checking graph connectivity.
10. Conductors of Probability. Min-entropy, its properties.
11. Zigzag construction for conductors.
12. Explicit construction of almost optimal bipartite expanders: use of spectral expanders and implicit constructions of size  $O(1)$ .
13. Zemor's codes. Encoding and decoding.
14. Codes on bipartite expanders. Encoding and decoding them.
15. Reliable storage of data in unreliable cells. Reliable boolean circuits.

- the mark "excellent (10)" is given to a student who has shown comprehensive, systematized, in-depth knowledge of the curriculum of the discipline and the ability to confidently apply them in practice when solving specific problems, free and correct justification of the decisions

- the mark "excellent (9)" is given to a student who has shown comprehensive, systematized, in-depth knowledge of the curriculum of the discipline and the ability to apply them in practice in solving specific problems, free and correct justification of the decisions

- the mark "excellent (8)" is given to a student who has shown comprehensive systematized, deep knowledge of the curriculum of the discipline and the ability to apply them in practice in solving specific problems, and the correct justification of the decisions made

- the mark "good (7)" is given to a student if he firmly knows the material, expresses it competently and to the point, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "good (6)" is given to a student if he knows the material, presents it competently and in essence, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "good (5)" is given to a student if he knows the material, and essentially expounds it, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "satisfactory (4)" is given to a student who has shown a fragmented, scattered nature of knowledge, insufficiently correct formulations of basic concepts, a violation of the logical sequence in the presentation of the program material, but at the same time he owns the main sections of the curriculum necessary for further education and can apply the obtained knowledge by model in a standard situation;

- the mark "satisfactory (3)" is given to a student who has shown a fragmented, scattered nature of knowledge, insufficiently correct formulations of basic concepts, violation of the logical sequence in the presentation of program material, but at the same time he knows fragmentarily the main sections of the curriculum necessary for further education and can apply the knowledge gained by the model in a standard situation;

- the mark "unsatisfactory (2)" is given to a student who does not know most of the main content of the curriculum of the discipline, makes gross mistakes in the formulation of the basic concepts of the discipline and does not know how to use the knowledge gained in solving typical practical problems

- grade "unsatisfactory (1)" is given to a student who does not know the formulations of the basic concepts of the discipline.

## **5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience**

During the differentiated assessment, students can use the discipline program.